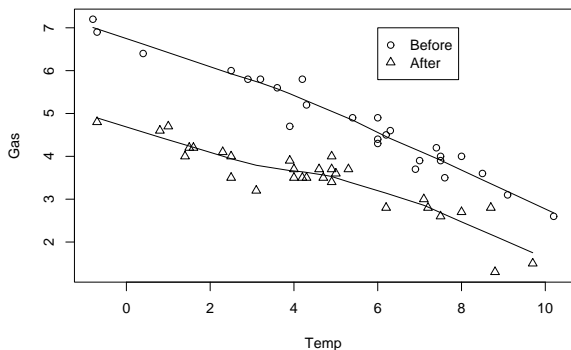# Gas Consumption

```
> library(MASS)
> names(whiteside)
[1] "Insul" "Temp"  "Gas"
> attach(whiteside)
> table(Insul)
Insul
Before  After
   26     30
> plot(Gas ~ Temp, pch=as.numeric(Insul))
> legend(6,7,c("Before","After"),pch=c(1,2))
> lines(lowess(Temp[Insul=="Before"],Gas[Insul=="Before"]))
> lines(lowess(Temp[Insul=="After"],Gas[Insul=="After"]))
>
```

Simple Linear Model: Before

```
> lb <- lm(Gas ~ Temp, data=whiteside, subset=Insul=="Before")
> summary(lb)
Call:
lm(formula = Gas ~ Temp, data = whiteside, subset = Insul ==
    "Before")
[ . . . ]
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.85383    0.11842   57.88   <2e-16 ***
Temp        -0.39324    0.01959  -20.08   <2e-16 ***
[ . . . ]
Residual standard error: 0.2813 on 24 degrees of freedom
Multiple R-Squared: 0.9438,Adjusted R-squared: 0.9415
F-statistic: 403.1 on 1 and 24 DF,  p-value: < 2.2e-16
> plot(lb)
```
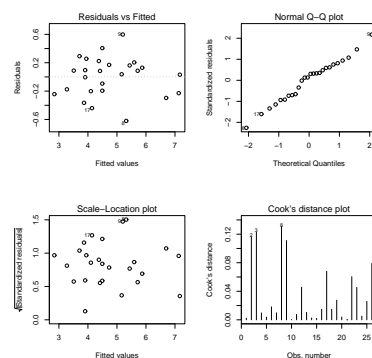
Simple Linear Model: After

```
> la <- update(lb, subset=Insul=="After")
> summary(la)
Call:
lm(formula = Gas ~ Temp, data = whiteside, subset = Insul ==
    "After")
[ . . . ]
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.72385    0.12974   36.41  < 2e-16 ***
Temp        -0.27793    0.02518  -11.04 1.05e-11 ***
[ . . . ]
Residual standard error: 0.3548 on 28 degrees of freedom
Multiple R-Squared: 0.8131,Adjusted R-squared: 0.8064
F-statistic: 121.8 on 1 and 28 DF,  p-value: 1.046e-11
> plot(la)
```
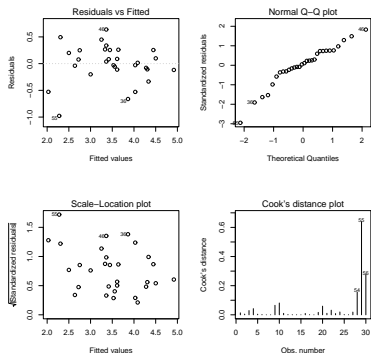
Comparison: Before and After

```
> coef(summary(la))
              Estimate Std. Error   t value     Pr(>|t|)
(Intercept)  4.7238497 0.12973942  36.41029 3.958906e-25
Temp        -0.2779350 0.02518429 -11.03605 1.045745e-11
> coef(summary(lb))
              Estimate Std. Error   t value     Pr(>|t|)
(Intercept)  6.8538277 0.11842341  57.87561 2.717533e-27
Temp        -0.3932388 0.01958601 -20.07754 1.640469e-16
> plot(Gas ~ Temp, pch=as.numeric(Insul), main="OLS Fits")
> legend(6,7,c("Before","After"),pch=c(1,2))
> abline(la); abline(lb)
```
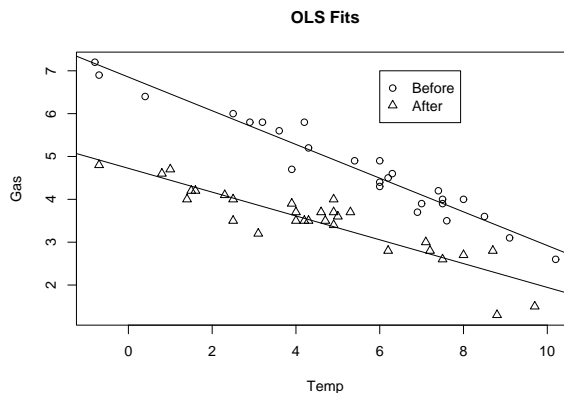
Combined Model

```
> lcomb <- lm(Gas ~ Insul/Temp - 1, data=whiteside)
> summary(lcomb)
Call:
lm(formula = Gas ~ Insul/Temp - 1, data = whiteside)
[ . . . ]
                 Estimate Std. Error t value Pr(>|t|)
InsulBefore       6.85383    0.13596   50.41   <2e-16 ***
InsulAfter        4.72385    0.11810   40.00   <2e-16 ***
InsulBefore:Temp -0.39324    0.02249  -17.49   <2e-16 ***
InsulAfter:Temp  -0.27793    0.02292  -12.12   <2e-16 ***
[ . . . ]
Residual standard error: 0.323 on 52 degrees of freedom
Multiple R-Squared: 0.9946,Adjusted R-squared: 0.9942
F-statistic:  2391 on 4 and 52 DF,  p-value: < 2.2e-16
> plot(lcomb)
```
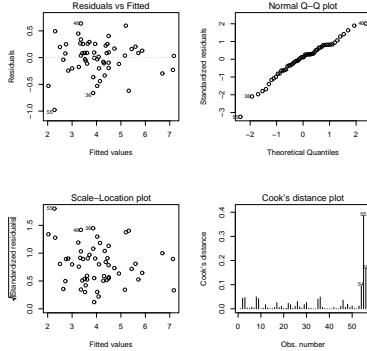
Comparison

```
> summary(lb)
[ . . . ]
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  6.85383    0.11842   57.88   <2e-16 ***
Temp        -0.39324    0.01959  -20.08   <2e-16 ***
[ . . . ]
Residual standard error: 0.2813 on 24 degrees of freedom
Multiple R-Squared: 0.9438,Adjusted R-squared: 0.9415
F-statistic: 403.1 on 1 and 24 DF,  p-value: < 2.2e-16
> summary(la)
[ . . . ]
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.72385    0.12974   36.41  < 2e-16 ***
Temp        -0.27793    0.02518  -11.04 1.05e-11 ***
[ . . . ]
Residual standard error: 0.3548 on 28 degrees of freedom
Multiple R-Squared: 0.8131,Adjusted R-squared: 0.8064
F-statistic: 121.8 on 1 and 28 DF,  p-value: 1.046e-11
> summary(lcomb)
[ . . . ]
                 Estimate Std. Error t value Pr(>|t|)
InsulBefore       6.85383    0.13596   50.41   <2e-16 ***
InsulAfter        4.72385    0.11810   40.00   <2e-16 ***
InsulBefore:Temp -0.39324    0.02249  -17.49   <2e-16 ***
InsulAfter:Temp  -0.27793    0.02292  -12.12   <2e-16 ***
[ . . . ]
Residual standard error: 0.323 on 52 degrees of freedom
Multiple R-Squared: 0.9946,Adjusted R-squared: 0.9942
F-statistic:  2391 on 4 and 52 DF,  p-value: < 2.2e-16
>
```

Analysis of Covariance

For $k \in \{1, 2\}$ corresponding to the "Before" and "After" groups, can the model

$$\text{Gas}_{k,j} = \alpha_k + \beta_k \text{Temp}_{k,j} + \epsilon_{k,j}$$

be replaced by the following simpler model?

$$\text{Gas}_{k,j} = \alpha_k + \beta \text{Temp}_{k,j} + \epsilon_{k,j}$$

We can fit the latter model by:

```
> lpara <- lm(Gas ~ Insul + Temp - 1, data=whiteside)
> summary(lpara)
[ . . . ]
            Estimate Std. Error t value Pr(>|t|)
InsulBefore  6.55133    0.11809   55.48   <2e-16 ***
InsulAfter   4.98612    0.10268   48.56   <2e-16 ***
Temp        -0.33670    0.01776  -18.95   <2e-16 ***
[ . . . ]
Residual standard error: 0.3574 on 53 degrees of freedom
Multiple R-Squared: 0.9933,Adjusted R-squared: 0.9929
F-statistic:  2600 on 3 and 53 DF,  p-value: < 2.2e-16
>
```

and compare two such "nested" models with:

```
> anova(lpara, lcomb)
Analysis of Variance Table

Model 1: Gas ~ Insul + Temp - 1
Model 2: Gas ~ Insul/Temp - 1
  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1     53 6.7704
2     52 5.4252  1    1.3451 12.893 0.0007307 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
```

An Alternate Parametrization

The alternative parametrization fit by:

```
> summary(lmalt <- lm(Gas ~ Insul*Temp, data=whiteside))
[ . . . ]
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      6.85383    0.13596  50.409  < 2e-16 ***
InsulAfter      -2.12998    0.18009 -11.827 2.32e-16 ***
Temp            -0.39324    0.02249 -17.487  < 2e-16 ***
InsulAfter:Temp  0.11530    0.03211   3.591  0.00073 ***
[ . . . ]
Residual standard error: 0.323 on 52 degrees of freedom
Multiple R-Squared: 0.9277,Adjusted R-squared: 0.9235
F-statistic: 222.3 on 3 and 52 DF,  p-value: < 2.2e-16
>
```

corresponds to the model:

$$\text{Gas}_j = \alpha_{\text{Before}} + \alpha_{\text{diff}} I_{\text{After},j} + \beta_{\text{Before}} \text{Temp}_j + \beta_{\text{diff}} I_{\text{After},j} \text{Temp}_j$$

where $I_{\text{After},j}$ is an indicator for "after insulation".
Note that the slope for "before insulation" is β_{Before}, and the slope for "after insulation" is $\beta_{\text{Before}} + \beta_{\text{diff}}$, so the fourth t-test corresponding to $H_0: \beta_{\text{diff}} = 0$ is a test of no difference in slopes before and after insulation.