Formulas

Arc Length

Arc length of curve $\vec{\mathbf{r}}(t)$, $a \le t \le b$ is

$$s = \int_a^b \left|\frac{d\vec{\mathbf{r}}}{dt}\right| dt = \int_a^b |\vec{\mathbf{v}}(t)| dt = \int_a^b v(t) dt$$

Hessian Test

$$H(x,y) = \begin{bmatrix} f_{11}(x,y) & f_{12}(x,y) \\ f_{12}(x,y) & f_{22}(x,y) \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}$$

- If $AD > B^2$ then
  - if $A < 0$ then local max;
  - if $A > 0$ then local min.
- If $AD < B^2$ then saddle point.

Newton's Method

$$\vec{\mathbf{r}}_n = \vec{\mathbf{r}}_{n-1} - \big(H(\vec{\mathbf{r}}_{n-1})\big)^{-1} \nabla f(\vec{\mathbf{r}}_{n-1})$$

Volumes

Volume between $z = z_{\text{top}}(x,y)$ and $z = z_{\text{bot}}(x,y)$ over domain D is

$$\iint_D \big(z_{\text{top}}(x,y) - z_{\text{bot}}(x,y)\big) dA$$

Integrals

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C \qquad (a > 0,\ |x| < a)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \qquad (a > 0)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C \qquad (a > 0)$$

Quadratic Approximation

$$f(x,y) \approx Q(x,y) = f(a,b) + f_1(a,b)(x-a) + f_2(a,b)(y-b) + \frac{f_{11}(a,b)}{2}(x-a)^2 + \frac{f_{22}(a,b)}{2}(y-b)^2 + f_{12}(a,b)(x-a)(y-b)$$

Chain Rules

$$\frac{d}{dt} f\Big(x(t), y(t)\Big) = f_1\Big(x(t), y(t)\Big) x'(t) + f_2\Big(x(t), y(t)\Big) y'(t)$$

$$\frac{\partial}{\partial u} f\Big(x(u,v), y(u,v)\Big) = f_1\Big(x(u,v), y(u,v)\Big) \frac{\partial}{\partial u} x(u,v) + f_2\Big(x(u,v), y(u,v)\Big) \frac{\partial}{\partial u} y(u,v)$$