Formulas

Projections

Scalar projection: $s = \dfrac{\vec{\mathbf{u}} \bullet \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|} = |\vec{\mathbf{u}}| \cos\theta$

Vector projection: $\vec{\mathbf{u}}_{\vec{\mathbf{v}}} = \dfrac{\vec{\mathbf{u}} \bullet \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|^2}\vec{\mathbf{v}} = \dfrac{\vec{\mathbf{u}} \bullet \vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}\hat{\mathbf{v}}$

Distances

Point (x_0, y_0, z_0) to plane $Ax + By + Cz = D$:

$$s = \frac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

Point $\vec{\mathbf{r}}_0 = \langle x_0, y_0, z_0 \rangle$ to line $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_1 + t\vec{\mathbf{v}}$:

$$s = \frac{|(\vec{\mathbf{r}}_0 - \vec{\mathbf{r}}_1) \times \vec{\mathbf{v}}|}{|\vec{\mathbf{v}}|}$$

Line $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_1 + t\vec{\mathbf{v}}_1$ to line $\vec{\mathbf{r}}(t) = \vec{\mathbf{r}}_2 + t\vec{\mathbf{v}}_2$:

$$s = \frac{|(\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1) \bullet (\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2)|}{|\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2|}$$

Arc Length

Arc length of curve $\vec{\mathbf{r}}(t)$, $a \le t \le b$:

$$s = \int_a^b \left|\frac{d\vec{\mathbf{r}}}{dt}\right| dt = \int_a^b |\vec{\mathbf{v}}(t)| dt = \int_a^b v(t) dt$$

Normals and Tangents to Surfaces

Normal vector to $z = f(x, y)$ at $(a, b, f(a, b))$:

$$\vec{\mathbf{n}} = \langle f_1(a, b), f_2(a, b), -1 \rangle$$

Tangent plane to $z = f(x, y)$ at $(a, b, f(a, b))$:

$$z = f(a, b) + f_1(a, b)(x - a) + f_2(a, b)(y - b)$$

Integrals

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C \qquad (a > 0,\ |x| < a)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \qquad (a > 0)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{x + a}{x - a}\right| + C \qquad (a > 0)$$