

Mathematics 263, Section 103 (Instructor Loewen)

Midterm 1

September 29, 2004

There are four questions worth a total of 100 marks.

No calculators or cheat sheets are allowed.

$\left[\frac{25}{100}\right]$ 1) Let \mathcal{P} be the plane containing the point $(2, 0, 0)$ and the line $2x + z = 2$, $2y + z = 2$.

Let \mathcal{L} be the line $\vec{\mathbf{r}}(t) = \langle -2, -1, -1 \rangle + t \langle 1, 0, -1 \rangle$.

$\left[\frac{10}{100}\right]$ a) Find the equation of \mathcal{P} .

$\left[\frac{5}{100}\right]$ b) Show that \mathcal{L} is parallel to \mathcal{P} .

$\left[\frac{10}{100}\right]$ c) Find the distance from \mathcal{L} to \mathcal{P} .

- $\left[\frac{25}{100}\right]$ 2) Consider the surface \mathcal{S} whose equation is $x^2 - 2x + y^2 - z^2 = -2$.
- $\left[\frac{10}{100}\right]$ a) Sketch \mathcal{S} , clearly explaining how you arrived at the sketch.
- $\left[\frac{7}{100}\right]$ b) Find the tangent plane to \mathcal{S} at the point $(0, 1, \sqrt{3})$.
- $\left[\frac{8}{100}\right]$ c) Parametrize the curve of intersection of \mathcal{S} with the plane $z = \sqrt{5}$.

$\left[\frac{25}{100}\right]$ 3) Define the function

$$f(x, y) = \begin{cases} \frac{x^2 - 2x + y^2 + y - 1}{x + y - 2} & \text{if } x + y \neq 2 \\ 3 & \text{if } x + y = 2 \end{cases}$$

$\left[\frac{12}{100}\right]$ a) Evaluate, if possible, $\frac{\partial f}{\partial y}(1, 2)$.

$\left[\frac{13}{100}\right]$ b) Evaluate, if possible, $\frac{\partial f}{\partial y}(1, 1)$.

- $\left[\frac{25}{100}\right]$ 4) Suppose that the temperature in degrees Centigrade at the point (x, y, z) , with coordinates measured in meters, is given by the function $T(x, y, z) = 20 + \sin(xyz + 6)$. A bee located at the point $P = (2, -1, 3)$ is flying towards the point $Q = (1, 1, 1)$.
- $\left[\frac{10}{100}\right]$ a) Find the directional derivative of $T(x, y, z)$ at P in the direction of the bee's motion.
- $\left[\frac{10}{100}\right]$ b) Estimate the temperature at the point $R = (2.1, -0.95, 3.25)$.
- $\left[\frac{5}{100}\right]$ c) In what direction should the bee move in order to warm up as quickly as possible?

Extra writing room