Math 263 HW07; Due 10 November 2004

1. Compute the volumes of the following regions.

(a) The "ice-cream cone" region which is bounded above by the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ and below by the cone $z = \sqrt{x^2 + y^2}$.

(b) The region bounded by $z = x^2 + 3y^2$ and $z = 4 - y^2$.

(c) The region inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cylinder $x^2 + y^2 = 4$.

2. Let T be the solid bounded by $z = 2$ and $z = \frac{1}{2}\sqrt{x^2 + y^2}$ and with $y \geq 0$. Furthermore, assume that T has constant density $\delta(x, y, z) = \alpha > 0$.

(a) Compute the center of mass, $(\overline{x}, \overline{y}, \overline{z})$, of T. Recall that

$$\overline{x} = \frac{1}{M}\int\int\int_T x\delta(x, y, z)dV; \quad \overline{y} = \frac{1}{M}\int\int\int_T y\delta(x, y, z)dV; \quad \overline{z} = \frac{1}{M}\int\int\int_T z\delta(x, y, z)dV$$

where $M =$ mass of $T = \int\int\int_T \delta(x, y, z)dV$.

(b) Verify the mass of T using SPHERICAL coordinates.

3. Let C be the helix $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$ for $0 \leq t \leq 2\pi$ and let $f(x, y, z) = x^2 + y^2 + z^2$.

(a) Compute the arc length, s, of C.

(b) Evaluate $\int_C f(x, y, z)ds$.

(c) Evaluate $\frac{1}{s}\int_C f(x, y, z)ds$, i.e., the average value of $f(x, y, z)$ along C.

4. Compute $\int_C f(x, y, z)ds$ for the following curves and functions.

(a) $C : \mathbf{r}(t) = \langle 30\cos^3 t, 30\sin^3 t \rangle$ for $0 \leq t \leq \pi/2$ and $f(x, y) = 1 + y/3$.

(b) $C : \mathbf{r}(t) = \langle t^2/2, t^3/3 \rangle$ for $0 \leq t \leq 1$ and $f(x, y) = x^2 + y^2$.

(c) $C : \mathbf{r}(t) = \langle 1, 2, t^2 \rangle$ for $0 \leq t \leq 1$ and $f(x, y, z) = e^{\sqrt{z}}$.

5. Determine whether or not the following vector fields are conservative. In the cases where $\mathbf{F}$ is conservative, find a function φ such that $\mathbf{F}(x, y, z) = \nabla\varphi(x, y, z)$.

(a) $\mathbf{F} = (2xy + z^2)\mathbf{i} + (x^2 + 2yz)\mathbf{j} + (y^2 + 2xz)\mathbf{k}$.

(b) $\mathbf{F} = (\ln(xy))\mathbf{i} + (\frac{x}{y})\mathbf{j} + (y)\mathbf{k}$.

(c) $\mathbf{F} = (e^x \cos y)\mathbf{i} + (-e^x \sin y)\mathbf{j} + (2z)\mathbf{k}$.

(d) $\mathbf{F} = (3x^2 y)\mathbf{i} + (4xy^2)\mathbf{j}$.

6. Let C_1 be the piece of the parabola $y = x^2$ from $P = (0, 0)$ to $Q = (1, 1)$ and let C_2 be the straight line from P to Q.

(a) Compute $\int_{C_1} y^2 dx + (x - y)dy$.

(b) Compute $\int_{C_2} y^2 dx + (x - y)dy$.

(c) Based on your answers to (a) and (b), is the vector field $\mathbf{F}(x, y) = \langle y^2, x - y \rangle$ conservative? Why or why not?

(d) Let $C = C_1 - C_2$ denote the end-to-end concatenation of C_1 with its given orientation and C_2 with the reverse orientation. Compute $\int_C xy^2 dx + x^2 y dy$.