Math 263 Assignment 6

Due in class on Wednesday 3 November 2004

1. Evaluate these iterated integrals: $\quad I = \int_0^2 \int_0^y y^2 e^{xy}\,dx\,dy, \quad J = \int_0^\pi \int_{-x}^x \cos y\,dy\,dx.$

2. For any reasonable $f\colon \mathbb{R}^2 \to \mathbb{R}$, let

$$I[f] = \int_{1/\sqrt{2}}^1 \int_{\sqrt{1-x^2}}^x f(x,y)\,dy\,dx + \int_1^{\sqrt{2}} \int_0^x f(x,y)\,dy\,dx + \int_0^{\sqrt{2}} \int_{\sqrt{2}}^{\sqrt{4-y^2}} f(x,y)\,dx\,dy.$$

(a) Find and sketch the set D in the xy-plane such that, for any f, $\quad I[f] = \iint_D f(x,y)\,dA.$

(b) Evaluate $I[f]$ for the constant function $f(x,y) = K$.

(c) Evaluate $I[f]$ for $f(x,y) = \sqrt{x^2+y^2}$.

3. For each iterated integral, sketch the domain of integration and evaluate:

$$I = \int_0^1 dy \int_y^1 e^{-x^2}\,dx, \qquad J = \int_0^{\pi/2} \int_y^{\pi/2} \frac{\sin x}{x}\,dx\,dy, \qquad K = \int_0^1 \int_x^1 \frac{y^p}{x^2+y^2}\,dy\,dx\ (p>0).$$

4. Find the volume of the solid under the surface $z = x^2 \sin(y^4)$ and above the triangle in the xy-plane whose vertices are $(0,0)$, $(0,\pi^{1/4})$, and $(\pi^{1/4},\pi^{1/4})$.

5. Let $s > 0$ be a constant, and consider the set $\quad D = \left\{(x,y) \in \mathbb{R}^2 \,:\, x \ge 0,\ 0 \le y \le e^{-sx}\right\}.$

Express the centroid coordinates $(\overline{x}, \overline{y})$ for D in terms of s. Recall that

$$\overline{x} = \frac{1}{\text{Area}(D)} \iint_D x\,dA, \qquad \overline{y} = \frac{1}{\text{Area}(D)} \iint_D y\,dA; \quad \text{in general,} \quad \overline{f} = \frac{1}{\text{Area}(D)} \iint_D f(x,y)\,dA.$$

6. Sketch the plane curves given by these polar equations:

(i) $r = 1 + \sin\theta$, (ii) $r = 1 + 2\cos\theta$, (iii) $r = \cos 3\theta$, (iv) $r^2 = 4\sin 3\theta$.

Work without electronic assistance as much as possible.
[Checking your results with a computer *after* thinking about the problem is strongly encouraged.]

7. Find $\iint_S x\,dA$, where S is the disk segment $x^2 + y^2 \le 2$, $x \ge 1$.

8. The disk D, $x^2 + y^2 \le 1$, is inscribed in a square whose sides have length 2. For each point (x,y) in D, let the distance to the nearest side of the square be $f(x,y)$. Find $\overline{f}$, the average value of f on D.

9. Find the volume of the region above the xy-plane, below the paraboloid $z = 1 - x^2 - y^2$, and in the wedge $-x \le y \le \sqrt{3}\,x$.

10. Evaluate these integrals by changing to spherical coordinates:

(a) $\displaystyle\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx.$

(b) $\displaystyle\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} \left(x^2+y^2+z^2\right)\,dz\,dx\,dy.$

11. Evaluate the improper integral $\quad Q = \iiint_{\mathbb{R}^3} |\mathbf{x}|\, e^{-|\mathbf{x}|^2}\,dV \quad$ by interpreting it as the limit when $R \to \infty$ of a triple integral over a solid sphere of radius R centred at $\mathbf{0}$.