Math 263 Assignment 5

Due October 20

1) If t_0 is a local minimum or maximum of the smooth function $f(t)$ of one variable (t runs over all real numbers) then $f'(t_0) = 0$. Derive an analogous necessary condition for $\vec{x}_0$ to be a local minimum or maximium of the smooth function $g(\vec{x})$ **restricted** to points on the line $\vec{x} = \vec{a} + t\vec{d}$. The test should involve the gradient of $g(\vec{x})$.

2) Find the maximum and minimum values of $f(x, y) = xy - x^3y^2$ when (x, y) runs over the square $0 \le x \le 1,\ 0 \le y \le 1$.

3) The temperature at all points in the disc $x^2 + y^2 \le 1$ is $T(x, y) = (x + y)e^{-x^2-y^2}$. Find the maximum and minimum temperatures at points of the disc.

4) Find the high and low points of the surface $z = \sqrt{x^2 + y^2}$ with (x, y) varying over the square $|x| \le 1, |y| \le 1$. Discuss the values of $z_x,\ z_y$ there. Do not evaluate any derivatives in answering this question.

5) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x + y - z$ on the sphere $x^2 + y^2 + z^2 = 1$.

6) Find $a,\ b$ and c so that the volume $4\pi abc/3$ of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ passing through the point $(1, 2, 1)$ is as small as possible.

7) Find the ends of the major and minor axes of the ellipse $3x^2 - 2xy + 3y^2 = 4$.

8) Find the triangle of largest area that can be inscribed in the circle $x^2 + y^2 = 1$.

9) The temperature gradient, at each point (x, y) of the disk $x^2 + y^2 \le 25$, is a strictly positive multiple of $(6 + x, 8 + y)$. Find the hottest point of the disk.

10) For each of the following, evaluate the given double integral **without** using iteration. Instead, interpret the integral as an area or some other physical quantity.

    a) $\iint_R dx\, dy$ where R is the rectangle $-1 \le x \le 3,\ -4 \le y \le 1$.

    b) $\iint_D (x + 3) dx\, dy$, where D is the half disc $0 \le y \le \sqrt{4 - x^2}$.

    c) $\iint_R (x + y) dx\, dy$ where R is the rectangle $0 \le x \le a,\ 0 \le y \le b$.

    d) $\iint_R \sqrt{a^2 - x^2 - y^2}\, dx\, dy$ where R is the region $x^2 + y^2 \le a^2$.

    e) $\iint_R \sqrt{b^2 - y^2}\, dx\, dy$ where R is the rectangle $0 \le x \le a,\ 0 \le y \le b$.