**Math 263 Assignment #3**

*Due in class on Wednesday 6 October 2004*

1. A function $z = f(x, y)$ is called *harmonic* if it satisfies Laplace's equation:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

Determine whether or not the following are harmonic.

(a) $z = \sqrt{x^2 + y^2}$.
(b) $z = e^{-x} \sin y$.
(c) $z = 3x^2 y - y^3$.

2. Give an example of a function with the indicated properties or show that no such function exists.

(a) A function $f(x, y)$ with continuous first and second order partial derivatives in the $xy$-plane and which satisfies $\partial f/\partial x = 6xy^2$ and $\partial f/\partial y = 8x^2 y$.
(b) A function $g(x, y)$ satisfying the equations $\partial g/\partial x = \partial g/\partial y = 2xy$.

3. Use the appropriate version of the chain rule to compute the following.

(a) $dw/dt$ at $t = 3$, where $w = \ln(x^2 + y^2 + z^2)$, $x = \cos t$, $y = \sin t$, and $z = 4\sqrt{t}$.
(b) $\partial z/\partial u$ and $\partial z/\partial v$, where $z = xy$, $x = u \cos v$, and $y = u \sin v$.

4. Suppose that a duck is swimming in the circle $x = \cos t$, $y = \sin t$ and that the water temperature is given by the formula $T = x^2 e^y - xy^3$. Find the rate of change in temperature the duck might feel.

5. A boat is sailing northeast at 20 km/h. Assuming that the temperature drops at a rate of 0.2°C/km in the northerly direction and 0.3°C/km in the easterly direction, what is the rate of change of temperature with respect to time as observed on the boat?

6. Compute the following using implicit differentiation.

(a) $\partial y/\partial z$ if $e^{yz} - x^2 z \ln y = \pi$.
(b) $dy/dx$ if $F(x, y, x^2 - y^2) = 0$.
(c) $(\partial y/\partial x)_u$ if $xyuv = 1$ and $x + y + u + v = 0$. [Hint: You should consider $y = y(x, u)$ and $v = v(x, u)$.]

7. Given a surface $F(x, y, z) = 0$ that defines $x = f(y, z)$, $y = g(x, z)$, and $z = h(x, y)$. Use implicit differentiation to verify that

$$\left(\frac{\partial y}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial x}{\partial z}\right) = -1.$$

8. Find second-degree (Taylor) polynomial approximations to the given functions at the points provided.

(a) $f(x, y) = \tan^{-1}(x + xy)$ at the point $(-1, 0)$.
(b) $g(x, y) = x^3 + 2xy + xy^2$ at the point $(1, 1)$.
(c) Use part (b) to estimate $(1.1)^3 + 2(1.1)(.9) + (1.1)(.9)^2$. [Note that the actual value is 4.202.]