# Math 263 Assignment 2

Due in class on Wednesday 22 September 2004

**1.** The position of a moving particle at time $t$ is given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$. Find

(a) The particle's velocity and acceleration vectors ($\mathbf{v}$ and $\mathbf{a}$) at $t = 1$;

(b) An equation for the plane through $\mathbf{r}(1)$ containing the two vectors in (a); and

(c) A pair of perpendicular unit vectors $\mathbf{u}$, $\mathbf{w}$ in the plane from (b), with $\mathbf{u} \parallel \mathbf{v}$.

**2.** The position of a moving particle at time $t$ is $\mathbf{r}(t) = \big(\cos(t), \sin(t), 2\cos^2(t)\big)$. Find all points on the particle's path at which its velocity and acceleration vectors are perpendicular. Illustrate with a good sketch.

**3.** A moving particle's position $\mathbf{r}(t)$ and velocity $\mathbf{v}(t)$ obey $\mathbf{v}(t) \perp \mathbf{r}(t)$ for all $t$.

(a) Prove that the particle must be moving on the surface of a sphere.

(b) Assuming that $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$, find the equation of the sphere in (a).

**4.** A pebble is placed at the origin and released with initial velocity $\mathbf{0}$. It slides without friction along the plane $3x - 2y - z = 0$. Find its position two seconds after release. [Assume a constant gravitational field of $-g\mathbf{k}$, where $g$ is a positive constant. Express your answer in terms of $g$.]

**5.** Given $\mathbf{r}(t) = 3(\sin t - t\cos t)\mathbf{i} + 3(\cos t + t\sin t)\mathbf{j} + 2t^2\mathbf{k}$, $t \geq 0$, find

(a) the length of the arc between $(0, 3, 0)$ and $(-6\pi, 3, 8\pi^2)$,

(b) the unit tangent $\widehat{\mathbf{T}} = \mathbf{v}/|\mathbf{v}|$ as a function of $t$, and

(c) a reparametrization of the same curve in terms of arc length (match $s = 0$ with $t = 0$).

**6.** Sketch and give the standard name for each surface below:

$$\text{(a)}\quad z = x^2 - 3y^2, \qquad \text{(b)}\quad x^2 + 4z^2 = 4, \qquad \text{(c)}\quad x^2 = y^2 + 2z^2.$$

**7.** Sketch the graphs of the functions $f(x, y) = 4 - x^2 - y^2$ and $g(x, y) = 4 - x^2$.

**8.** An object moves along the curve $y = x^2$, $z = x^3$, with constant vertical speed $dz/dt = 3$. Find the object's velocity and acceleration when it is at the point $(2, 4, 8)$. [Adams 5/e, 11.1 #18]

**9.** A duck flies with constant speed, 18 units/s, along the curve where the surfaces $y = x^2/3$ and $z = (2/9)xy$ meet. As the duck passes through the point $P(3, 3, 2)$, its $x$-coordinate is increasing. Find the duck's velocity and acceleration at $P$.

**10.** Write $S$ for the sphere $x^2 + (y-1)^2 + (z+2)^2 = 9$, and $P(k)$ for the plane $2x + 6y + 3z = k$.

(a) Find $c > 0$ such that $P(k)$ and $S$ meet in exactly one point if $k = \pm c$. Use $c$ in parts (b)–(c).

(b) When $|k| < c$, $P(k)$ meets $S$ in a circle. Express this circle's centre and radius in terms of $k$.

(c) Assuming $|k| < c$, find $\mathbf{r}_0$, $\mathbf{u}$, and $\mathbf{w}$ so that the circle of intersection between $P(k)$ and $S$ is given by

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{u}\cos(t) + \mathbf{w}\sin(t), \quad t \in \mathbb{R}, \qquad \text{with} \quad \mathbf{u} \parallel \langle -3, 0, 2 \rangle\,.$$