Math 263 Assignment 1

Due September 15

1) Find the equation of a sphere if one of its diameters has end points $(2, 1, 4)$ and $(4, 3, 10)$.

2) Show that the set of all points P that are twice as far from $(-1, 5, 3)$ as from $(6, 2, -2)$ is a sphere. Find its centre and radius.

3) Describe and sketch the set of all points in $\mathbb{R}^3$ that satisfy
   a) $x^2 + y^2 + z^2 = 2z$ b) $x^2 + z^2 = 4$ c) $z \geq \sqrt{x^2 + y^2}$
   d) $x^2 + y^2 + z^2 = 4,\ z = 1$ e) $x + y + z = 1$

4) The pressure $p(x, y)$ at the point (x, y) is determined by $x^2 - 2px + y^2 + 1 = 0$. Sketch the isobars (curves of constant pressure).

5) Compute the dot product of the vectors $\vec{a}$ and $\vec{b}$. Find the angle between them.
   a) $\vec{a} = \langle -1, 1 \rangle,\ \vec{b} = \langle 1, 1 \rangle$ b) $\vec{a} = \langle 1, 1 \rangle,\ \vec{b} = \langle 2, 2 \rangle$
   c) $\vec{a} = \langle 1, 2, 1 \rangle,\ \vec{b} = \langle -1, 1, 1 \rangle$

6) Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

7) Drop a perpendicular from the point $(6, 5, 1)$ to the line, L, through the points $(1, 2, 0)$ and $(3, 4, 6)$. Where does the perpendicular hit L?

8) Use a projection to derive a formula for the distance from a point (x_1, y_1) to the line $ax + by = c$. Here, a and b are not both zero.

9) Compute $\langle 1, 2, 3 \rangle \times \langle 4, 5, 6 \rangle$.

10) Prove that
   a) $\hat{\boldsymbol{\imath}} \times \hat{\boldsymbol{\jmath}} = \hat{\mathbf{k}}$
   b) $\vec{a} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{a} \times \vec{b}) = 0$
   c) $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 \, |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

11) Find the equation of the sphere which has the two planes $x+y+z = 3,\ x+y+z = 9$ as tangent planes if the centre of the sphere is on the planes $2x - y = 0,\ 3x - z = 0$.

12) Find the equation of the plane that passes through the point $(-2, 0, -1)$ and through the line of intersection of $2x + 3y - z = 0,\ x - 4y + 2z = -5$.

13) Find the equations of the line through $(2, -1, -1)$ and parallel to each of the two planes $x + y = 0$ and $x - y + 2z = 0$. Express the equations of the line in vector and scalar parametric forms and in symmetric form.