Marks

[15]   **1.**   The internal temperature of a solid ellipsoid $x^2 + xz + \frac{3}{2}z^2 + 2(y-2)^2 \leq 11$ is given by

$$T(x,y,z) = \sqrt{y+3}\, e^{2x-z}$$

(a)   Find a line that is perpendicular to the surface of the ellipsoid and passes through the point $P = (1,1,2)$. Call this line L.

(b)   Calculate the rate of temperature change per unit distance at P in the direction inward along L.

(c)   Estimate the temperature of the solid 0.09 units from point P inward along L.

[15] **2.** The mass m of an object with kinetic energy E and velocity v is $m = 2E/v^2$. If a body has a measured kinetic energy of 200 and a measured velocity of 100, but the measurements could have an error of $\pm 1\%$, what is the approximate maximum percentage error in the measurement of the mass?

[15]   **3.**   Find the minimum and maximum values of $x^2 + 2y^2 - x$ in the region $x^2 + y^2 \leq 1$.

[15] **4.**   Convert to polar coordinates and evaluate:

$$I = \int_0^2 \int_0^{\sqrt{2x-x^2}} \left(k + 3\sqrt{x^2 + y^2} \right) dy\, dx.$$

Express your answer in terms of the constant k.

[15] **5.** Let $\vec{\mathbf{F}}(x, y) = (\sin y + y \cos x)\hat{\boldsymbol{\imath}} + (\sin x + x \cos y)\hat{\boldsymbol{\jmath}}$.

    (a) Determine whether or not $\vec{\mathbf{F}}$ is conservative. If it is, find a potential function for $\vec{\mathbf{F}}$.

    (b) Calculate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where C is the piece of the parabola $y = \frac{2}{\pi}x^2$ from $A = (0, 0)$ to $B = \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

[15]   **6.**   Let D be the solid that is bounded below by the plane $2x + 2y + z + 2 = 0$ and is bounded above by the paraboloid $z = 4 - (x+1)^2 - (y+1)^2$. Let the field $\vec{\mathbf{F}}$ be given by

$$\vec{\mathbf{F}}(x, y, z) = \frac{\langle y, 1, z \rangle}{\sqrt{x^2 + y^2}}$$

(a)   Parameterize the curve of intersection of the plane and paraboloid in terms of the polar coordinate θ.

(b)   Let S_1 be the portion of the surface of D formed by the paraboloid. Parameterize S_1.

(c)   Let J denote the flux through S_1 into the solid D. Express J as an iterated double integral using the parameterization of part (b). Simplify the integrand, but **do not** evaluate this double integral.

[15] **7.** Let $\mathcal{R}$ denote the solid region defined by the simultaneous inequalities

$$x \geq 0, \quad y \geq 0, \quad z \geq 0, \quad 1 \leq x^2 + y^2 + z^2 \leq 4$$

Let $\mathcal{S}$ denote the surface of $\mathcal{R}$.

(a) Sketch $\mathcal{R}$ and $\mathcal{S}$.

(b) Evaluate the outward flux of the following vector field through $\mathcal{S}$:

$$\vec{\mathbf{F}}(x, y, z) = \left\langle x^5 + y\sin(z),\ y^5 + z\sin(x),\ 10x^2 y^2 z - x \right\rangle.$$

(c) Find the flux of $\vec{\mathbf{F}}$ downward through the bottom of $\mathcal{S}$, i.e., through the flat part of $\mathcal{S}$ that lies in the plane $z = 0$.

[15] **8.** Let $\vec{\mathbf{F}}(x, y, z) = (e^{x^2} + y)\,\hat{\boldsymbol{\imath}} + \left(\sin(y^3) + xz\right)\hat{\boldsymbol{\jmath}} + z^2\,\hat{\mathbf{k}}$. Use Stokes's theorem to evaluate $\int_{\mathcal{C}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where $\mathcal{C}$ is the curve $x^2 + y^2 = 10$, $x + y + z = 4$ with positive orientation (i.e. counter–clockwise) as viewed from high on the z–axis.

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Be sure that this examination has 12 pages including this cover

The University of British Columbia

Sessional Examinations - April 2005

Mathematics 263

Multivariable and Vector Calculus

Closed book examination   Time: $2\frac{1}{2}$ hours

Name —————————————   **Signature** —————————————

Student Number————————   **Instructor's Name** ————————

                                                       Section Number ————————

Special Instructions:

To receive full credit, all answers must be supported by clear and correct derivations.

No calculators, notes, or other aids are allowed. A formula sheet is provided with the exam.

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2		15
3		15
4		15
5		15
6		15
7		15
8		15
Total		120