

Marks

- [15] 1. The internal temperature of a solid ellipsoid $x^2 + xz + \frac{3}{2}z^2 + 2(y - 2)^2 \leq 11$ is given by

$$T(x, y, z) = \sqrt{y + 3} e^{2x - z}$$

- (a) Find a line that is perpendicular to the surface of the ellipsoid and passes through the point $P = (1, 1, 2)$. Call this line L .
- (b) Calculate the rate of temperature change per unit distance at P in the direction inward along L .
- (c) Estimate the temperature of the solid 0.09 units from point P inward along L .

- [15] **2.** The mass m of an object with kinetic energy E and velocity v is $m = 2E/v^2$. If a body has a measured kinetic energy of 200 and a measured velocity of 100, but the measurements could have an error of $\pm 1\%$, what is the approximate maximum percentage error in the measurement of the mass?

- [15] **3.** Find the minimum and maximum values of $x^2 + 2y^2 - x$ in the region $x^2 + y^2 \leq 1$.

- [15] 4. Convert to polar coordinates and evaluate:

$$I = \int_0^2 \int_0^{\sqrt{2x-x^2}} \left(k + 3\sqrt{x^2 + y^2} \right) dy dx.$$

Express your answer in terms of the constant k .

[15] **5.** Let $\vec{\mathbf{F}}(x, y) = (\sin y + y \cos x)\hat{\mathbf{i}} + (\sin x + x \cos y)\hat{\mathbf{j}}$.

- (a) Determine whether or not $\vec{\mathbf{F}}$ is conservative. If it is, find a potential function for $\vec{\mathbf{F}}$.
- (b) Calculate $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where C is the piece of the parabola $y = \frac{2}{\pi}x^2$ from $A = (0, 0)$ to $B = (\frac{\pi}{2}, \frac{\pi}{2})$.

- [15] **6.** Let D be the solid that is bounded below by the plane $2x + 2y + z + 2 = 0$ and is bounded above by the paraboloid $z = 4 - (x + 1)^2 - (y + 1)^2$. Let the field $\vec{\mathbf{F}}$ be given by

$$\vec{\mathbf{F}}(x, y, z) = \frac{\langle y, 1, z \rangle}{\sqrt{x^2 + y^2}}$$

- (a) Parameterize the curve of intersection of the plane and paraboloid in terms of the polar coordinate θ .
- (b) Let S_1 be the portion of the surface of D formed by the paraboloid. Parameterize S_1 .
- (c) Let J denote the flux through S_1 into the solid D . Express J as an iterated double integral using the parameterization of part (b). Simplify the integrand, but **do not** evaluate this double integral.

- [15] 7. Let \mathcal{R} denote the solid region defined by the simultaneous inequalities

$$x \geq 0, \quad y \geq 0, \quad z \geq 0, \quad 1 \leq x^2 + y^2 + z^2 \leq 4$$

Let \mathcal{S} denote the surface of \mathcal{R} .

- (a) Sketch \mathcal{R} and \mathcal{S} .
(b) Evaluate the outward flux of the following vector field through \mathcal{S} :

$$\vec{\mathbf{F}}(x, y, z) = \langle x^5 + y \sin(z), y^5 + z \sin(x), 10x^2y^2z - x \rangle.$$

- (c) Find the flux of $\vec{\mathbf{F}}$ downward through the bottom of \mathcal{S} , i.e., through the flat part of \mathcal{S} that lies in the plane $z = 0$.

- [15] 8. Let $\vec{\mathbf{F}}(x, y, z) = (e^{x^2} + y)\hat{\mathbf{i}} + (\sin(y^3) + xz)\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$. Use Stokes's theorem to evaluate $\int_{\mathcal{C}} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where \mathcal{C} is the curve $x^2 + y^2 = 10$, $x + y + z = 4$ with positive orientation (i.e. counter-clockwise) as viewed from high on the z -axis.

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The University of British Columbia

Sessional Examinations - April 2005

Mathematics 263

Multivariable and Vector Calculus

Closed book examination

Time: $2\frac{1}{2}$ hours

Name _____ Signature _____

Student Number _____ Instructor's Name _____

Section Number _____

Special Instructions:

To receive full credit, all answers must be supported by clear and correct derivations.

No calculators, notes, or other aids are allowed. A formula sheet is provided with the exam.

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1. Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

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Total		120