Math 263 Sample Final Exam

1. (a) Find a tangent vector to the curve of intersection of the surfaces $z = 3x^2 - y^2$ and $2x^2 + 2y^2 - z^2 = 0$ at $(1, 1, 2)$.
   (b) Find an equation for the tangent line of the above curve at the point $(1, 1, 2)$.

2. The temperature at (x, y, z) is given by $T(x, y, z) = x^2 - y^2 + z^2(x^2 + 1)$.
   (a) A heat-seeking mosquito at $(0, 3, 4)$ wishes to warm up as quickly as possible. In which direction should it head?
   (b) If the mosquito flies with speed 7, what rate of change of temperature does it experience in (a)?

3. Find the maximum and minimum values of $x + 2y - 3z$ over the solid ellipsoid given by $x^2 + 4y^2 + 9z^2 \leq 108$.

4. Let $f(x, y) = x^2y^2(5 - x - y)$. Apply one iteration of Newton's Method for finding critical points using an initial guess $\vec{\mathbf{x}}_0 = \langle 1, 1 \rangle$.

5. Consider the following force field, in which m, n, p, and q are constants:
$$\vec{\mathbf{F}} = (mxyz + z^2 - ny^2)\vec{\mathbf{i}} + (x^2z - 4xy)\vec{\mathbf{j}} + (x^2y + pxz + qz^3)\vec{\mathbf{k}}$$
   (a) Find all values of m, n, p, and q such that $\vec{\mathbf{F}}$ is conservative.
   (b) For every possible choice of m, n, p, and q in (a), find the work done by $\vec{\mathbf{F}}$ in moving a particle from the bottom to the top of the sphere $x^2 + y^2 + z^2 = 2z$.

6. Let a and b be positive constants, and let $\mathcal{S}$ be the part of the conical surface
$$a^2z^2 = b^2(x^2 + y^2)$$
   where $0 \leq z \leq b$. Evaluate the surface integral
$$I = \iint_{\mathcal{S}} (x^2 + y^2)\, dS$$

7. Let $\mathcal{S}$ be the surface
$$x^2 + y^2 + 2(z - 1)^2 = 6, \qquad z \geq 0$$
   oriented by the outward normal. Define
$$\vec{\mathbf{G}} = \nabla \times \vec{\mathbf{F}}, \qquad \text{where} \quad \vec{\mathbf{F}} = (xz - y^3 \cos z)\vec{\mathbf{i}} + x^3e^z\vec{\mathbf{j}} + xyze^{x^2+y^2+z^2}\vec{\mathbf{k}}$$
   Find $\displaystyle\iint_{\mathcal{S}} \vec{\mathbf{G}} \bullet d\vec{\mathbf{S}}$. [Hint: For one possible solution method, you may find it helpful to use the fact that $\displaystyle\int_0^{2\pi} \sin^4 x\, dx = \int_0^{2\pi} \cos^4 x\, dx = \frac{3\pi}{4}$.]

8. Let $\mathcal{R}$ be the part of the solid cylinder $x^2 + (y - 1)^2 \leq 1$ satisfying $0 \leq z \leq y$; let $\mathcal{S}$ be the boundary of $\mathcal{R}$. Given $\vec{\mathbf{F}} = x^2\vec{\mathbf{i}} + 2y\vec{\mathbf{j}} - 2z\vec{\mathbf{k}}$,
   (a) Find the total flux of $\vec{\mathbf{F}}$ outward through $\mathcal{S}$.
   (b) Find the total flux of $\vec{\mathbf{F}}$ outward through the (vertical) cylindrical sides of $\mathcal{S}$.