Math 221 (101) Matrix Algebra September 18, 2002

Quiz #2

DUE: Wednesday, September 25 at 9:00am

Rules:

1. Quizzes are due at the beginning of class on the due date. Late quizzes will not receive credit. Remember to include your name and student number on your answers.
2. You may work with other Math 221 students and use materials such as textbooks, notes, calculators, or computers. However, you must write up your own original, complete solutions. Copying all or part of an answer is not allowed.
3. Show all work. Explain your solutions clearly. You may write in pencil if you'd like.
4. The maximum possible score is 20.

Solve the following five problems, showing all work. Don't forget to put your name and student number on your answers.

1. Consider the matrix equation

$$\begin{bmatrix} 1 & -3 & -1 & 7 \\ 0 & 0 & 2 & -4 \\ 2 & -6 & -1 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(a) Is this equation homogeneous? Is it consistent? Explain your answers.

(b) Find the set of all solutions of this equation. Express your answer both as a general solution and in parametric vector form.

(c) Verify the following, showing your work:

$$\begin{bmatrix} 1 & -3 & -1 & 7 \\ 0 & 0 & 2 & -4 \\ 2 & -6 & -1 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 7 \end{bmatrix}$$

(d) Using the answers from parts (b) and (c), express the solution set of the following equation in parametric vector form:

$$\begin{bmatrix} 1 & -3 & -1 & 7 \\ 0 & 0 & 2 & -4 \\ 2 & -6 & -1 & 12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 7 \end{bmatrix}$$

2. For each of the following sets of vectors, determine if the set is linearly dependent or independent. For each linearly dependent set, express one of the vectors as a linear combination of the others.

(a) $\mathbf{u} = (1, 0, 2)$, $\mathbf{v} = (0, -1, -1)$, $\mathbf{w} = (0, 0, 0)$

(b) $\mathbf{a}_1 = (0, 4, 1, 8)$, $\mathbf{a}_2 = (1, 0, 1, 4)$

(c) $\mathbf{u}_1 = (1, 4)$, $\mathbf{u}_2 = (-1, 2)$, $\mathbf{u}_3 = (3, 6)$

3. Suppose we have a matrix equation $A\mathbf{x} = \mathbf{b}$ where the coefficient matrix A is a 50×50 matrix.

(a) If A has 20 pivot columns but the augmented matrix $\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$ has *more* than 20 pivot columns, how many pivot columns must the augmented matrix have? Is the matrix equation consistent or inconsistent? Why?

(b) If A has 50 pivot positions, what is the size of the solution set of the equation? Why?

4. Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ be vectors in $\mathbb{R}^5$. Suppose they satisfy the equation:

$$2\mathbf{v}_1 - \mathbf{v}_2 + \mathbf{v}_4 = 0$$

(a) Express $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

(b) Show that every linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ can be written as a linear combination of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ only. Use this to prove that $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ and $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are the same set.

(c) (Bonus question) If the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent, are $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ and $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ the same set? Why or why not? [Hint: Try to show that $\mathbf{v}_3$ can't be written as a linear combination of the other three vectors.]

5. Let A be the matrix given by:

$$A = \begin{bmatrix} 1 & h & 2 \\ 3 & 4 & k \end{bmatrix}$$

(a) For what values of h and k do the columns of A *not* span $\mathbb{R}^2$?

(b) For the h and k calculated in (a), determine the span of the columns of A. Give an example of a vector in $\mathbb{R}^2$ that is *not* in this span.