

Quiz #1

DUE: Wednesday, September 18 at 9:00am

Rules:

1. Quizzes are due at the beginning of class on the due date. Late quizzes will not receive credit. Remember to include your name and student number on your answers.
2. You may work with other Math 221 students and use materials such as textbooks, notes, calculators, or computers. However, you must write up your own original, complete solutions. Copying all or part of an answer is not allowed.
3. Show all work. Explain your solutions clearly. You may write in pencil if you'd like.
4. The maximum possible score is 20.

Solve the following five problems, showing all work. Don't forget to put your name and student number on your answers.

1. Consider the following linear system:

$$\begin{cases} x_1 + x_2 - x_4 = 3 \\ x_2 + 2x_3 = -1 \\ x_1 - x_2 - 4x_3 - 2x_4 = 6 \end{cases}$$

- (a) Write down the augmented matrix of this system.
 - (b) Use the row reduction algorithm (or another sequence of elementary row operations) to reduce this matrix to its reduced echelon form. State the row operations you are using, and show all intermediate matrices.
 - (c) Write down the general solution of the system. What is the size of the solution set?
 - (d) Find the set of all solutions of this system that also satisfy $x_1 = 0$.
2. Suppose you are given a system of 1,000 equations in 900 variables.
 - (a) What is the size of the coefficient matrix? the augmented matrix?

A computer program reduces the augmented matrix to an echelon form with 901 non-zero rows.¹

- (b) How many pivot columns does the augmented matrix have?
- (c) What can you say about the solution set of the original system?

¹Systems this big are easily solved by computer. One student in this class wrote a computer program in C++ that took 20 seconds to reduce a system of this size to echelon form and another 8 seconds to reduce it to reduced echelon form, even on a fairly slow computer (with an Intel Celeron 300A processor running at 450MHz).

3. For each of the following matrix-vector products, calculate its value or state that it is undefined:

$$(a) \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} = ? \quad (b) \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \\ 8 \end{bmatrix} = ?$$

$$(c) \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = ? \quad (d) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = ?$$

4. Consider the two vectors:

$$\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

- (a) Calculate the linear combination $\mathbf{y} = 2\mathbf{u} - \mathbf{v}$.
 (b) Determine if the vector $\mathbf{w} = (15, 1, 11)$ is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$.
5. Fertilizer has an NPK rating indicating the amount of nitrogen (atomic), phosphate (P_2O_5), and potassium (as K_2O) it contains as a percentage by weight. For example, each gram of fertilizer with an NPK rating of 5–10–15 contains 0.05 grams of nitrogen, 0.10 grams of phosphate, and 0.15 grams of potassium.

You're an industrial chemist at FERTILIZERS-R-US, and your boss informs you the company has just gotten a great deal on some generic garden fertilizer (NPK 15–30–15) and a truckload of potash (NPK 15–0–45). He's very excited about blending these two components together to make new fertilizers the company can sell for a big profit.

- (a) Express the garden fertilizer as a vector giving the nitrogen, phosphate, and potassium content in each gram. Do the same for the potash.
 (b) If you mixed 25 grams of garden fertilizer with 75 grams of potash, what would be the nitrogen, phosphate, and potassium content of the result? Express your answer as a linear combination of the two vectors from part (a), and calculate its value.
 (c) In the context of this problem, what would it mean if a particular vector was *not* in the span of the two vectors you calculated in part (a)?