Math 221 (101) Matrix Algebra　　　　September 18, 2002

Quiz #1 Solutions

1. (a) $\begin{bmatrix} 1 & 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 0 & -1 \\ 1 & -1 & -4 & -2 & 6 \end{bmatrix}$

(b)

$$\xrightarrow{R3\to R3-R1} \begin{bmatrix} 1 & 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & -2 & -4 & -1 & 3 \end{bmatrix} \xrightarrow{R3\to R3+2R2} \begin{bmatrix} 1 & 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R3\to -R3} \begin{bmatrix} \boxed{1} & 1 & 0 & -1 & 3 \\ 0 & \boxed{1} & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{R1\to R1+R3} \begin{bmatrix} \boxed{1} & 1 & 0 & 0 & 2 \\ 0 & \boxed{1} & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow{R1\to R1-R2} \begin{bmatrix} \boxed{1} & 0 & -2 & 0 & 3 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & \boxed{1} & -1 \end{bmatrix}$$

(c) $\begin{cases} x_1 = 3 + 2x_3 \\ x_2 = -1 - 2x_3 \\ x_4 = -1 \\ x_3 \text{ free} \end{cases}$

Because there is a free variable, the size of the solution set is *infinite*.

(d) Substitute $x_1 = 0$ into the first equation of the general solution: $0 = 3 + 2x_3$, so $x_3 = -3/2$.

Then, the general solution gives a specific solution for $x_3 = -3/2$:

$$\begin{cases} x_1 = 3 + 2(-3/2) = 0 \\ x_2 = -1 - 2(-3/2) = 2 \\ x_4 = -1 \\ x_3 = -3/2 \end{cases}$$

Therefore, the set of all solutions with $x_1 = 0$ is the single, unique solution

$$\begin{cases} x_1 = 0 \\ x_2 = 2 \\ x_3 = -3/2 \\ x_4 = -1 \end{cases}$$

2. (a) The coefficient matrix is 1000×900. The augmented matrix is 1000×901.

(b) There are 901 pivot positions (the leading entries of the nonzero rows of the echelon matrix). Each pivot position is in a separate column. Therefore, there are 901 pivot columns in the augmented matrix.

(c) By (b), *all* 901 columns of the augmented matrix are pivot columns, including the rightmost column. Therefore, the system is inconsistent, and the solution set is empty.

3. (a)
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} = a \begin{bmatrix} 1 \\ 4 \end{bmatrix} + b \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 0 \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} a \\ 4a \end{bmatrix} + \begin{bmatrix} 2b \\ 5b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a + 2b \\ 4a + 5b \end{bmatrix}$$

(b) $$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \\ 8 \end{bmatrix} = 1[0] + 2[0] + 0[0] + (-1)[1] + 8[0] = [-1]$$

Don't forget the square brackets here! That $[-1]$ *is still a vector.*

(c) $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is undefined. There are 3 weights in the vector but only 2 columns in the matrix.

(d) $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

4. (a)
$$\mathbf{y} = 2\mathbf{u} - \mathbf{v} = 2 \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -1 \end{bmatrix}$$

(b) We need to know if $x_1\mathbf{u} + x_2\mathbf{v} = \mathbf{w}$ has a solution. The augmented matrix is

$$\begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 15 \\ 3 & -1 & 1 \\ 0 & 1 & 11 \end{bmatrix}$$

Row reducing,

$$\xrightarrow{R2 \to R2 - 3R1} \begin{bmatrix} 1 & 1 & 15 \\ 0 & -4 & -44 \\ 0 & 1 & 11 \end{bmatrix} \overset{\text{optional, but seems easier to me}}{\overbrace{\xrightarrow{R2 \leftrightarrow R3}}} \begin{bmatrix} 1 & 1 & 15 \\ 0 & 1 & 11 \\ 0 & -4 & -44 \end{bmatrix}$$
$$\Uparrow \qquad\qquad\qquad \Uparrow$$

$$\xrightarrow{R3 \to R3+4R2} \begin{bmatrix} 1 & 1 & 15 \\ 0 & 1 & 11 \\ 0 & 0 & 0 \end{bmatrix}$$
$$\Uparrow$$

The system is consistent. Therefore $x_1\mathbf{u} + x_2\mathbf{v} = \mathbf{w}$ has a solution. Therefore, yes, $\mathbf{w} \in \mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$.

5. (a) Garden fertilizer: $\begin{bmatrix} 0.15 \\ 0.30 \\ 0.15 \end{bmatrix}$ (0.15g of nitrogen) (0.30g of phosphate) (0.15g of potassium)

Potash: $\begin{bmatrix} 0.15 \\ 0 \\ 0.45 \end{bmatrix}$

(b) $$25 \begin{bmatrix} 0.15 \\ 0.30 \\ 0.15 \end{bmatrix} + 75 \begin{bmatrix} 0.15 \\ 0 \\ 0.45 \end{bmatrix} = \begin{bmatrix} 3.75 \\ 7.5 \\ 3.75 \end{bmatrix} + \begin{bmatrix} 11.25 \\ 0 \\ 33.75 \end{bmatrix} = \begin{bmatrix} 15 \\ 7.5 \\ 37.5 \end{bmatrix}$$

That is, 15g of nitrogen, 7.5g of phosphate, and 37.5g of potassium.

(c) If a vector isn't in the span of the vectors in part (a), it's not a linear combination of them.

In the context of this problem, that means we *can't* blend the garden fertilizer and the potash together in the right proportions to make a blended ferilizer with the nitrogen, phosphate, and potassium content given by the vector.