Principal Axes of a Quad. Form

Let $A = \begin{bmatrix} 13 & -4 \\ -4 & 7 \end{bmatrix}$. This can be orthogonally diagonalized:

$$A = \underbrace{\begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 5 & 0 \\ 0 & 15 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}}_{P^T}$$

and the principal axes are given by:

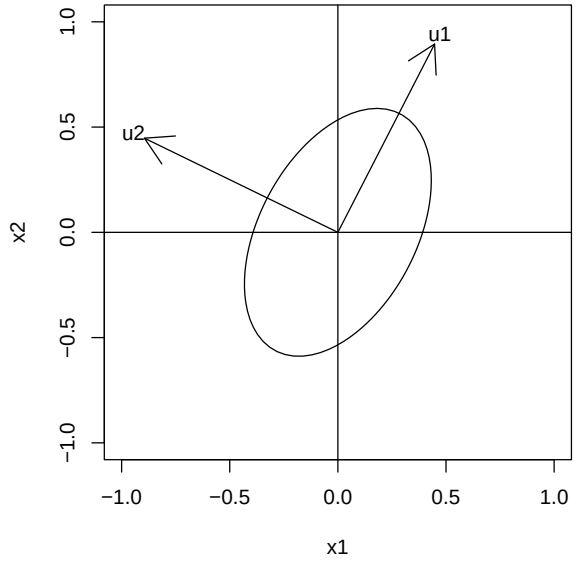
$$\mathbf{u}_1 = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix} \qquad \mathbf{u}_2 = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

Solution Set of a Quadratic Form

Fix $c = 2$. Then, the solution set of

$$Q(\mathbf{x}) = \mathbf{x}^T A \mathbf{x} = c$$

is the curve:



(May also be empty, a point, one or two lines, or a hyperbola.)

In Principal Axis Coordinates